

ON NECESSITY AND EXISTENCE

GUILLERMO E. ROSADO HADDOCK

At the end of our paper published in *Nominazione*¹ we expressed some doubts concerning Kripke's characterization of the necessity of statements containing denotative expressions.² In this paper we will be concerned with a detailed discussion of such doubts.

§1 If we take as our starting point the world actually existing, we can conceive of different possible worlds as distinct worlds that would be actual if some of the states of affairs holding in the actual world did not hold, whereas some other states of affairs incompatible with them did hold. With respect to statements we could say that in the different possible worlds some of the statements true in the actual world would be false, whereas some statements false in the actual world would be true.

Intuitively, statements that are true in the actual world, but false in other possible worlds are only contingently true, whereas statements that are false in the actual world but true in other possible worlds are only contingently false but (logically) possible. It would, thus, seem natural to call a statement 'logically necessary' -or simply 'necessary'- if it is true in every possible world. Such intuitive notions, however, are not free from difficulties. If in a possible world the denotative expressions occurring in a statement do not have any referent, it does not seem clear at all if we are to consider such a statement as true, as false, or as devoid of truth value in that possible world. Thus, e.g., although it seems clear that the statement 'Pegasus is a winged horse' should be true in any pos-

¹ 'Necessità a posteriori e Contingenze a priori in Kripke: Alcune Note Critiche', in *Nominazione* 2, June 1981.

² I.e., expressions that are supposed to have reference. For our purposes we can assume that only proper names and definite descriptions are denotative expressions.

sible world in which Pegasus exists (since Pegasus per definitonem is a winged horse) -and that the statement 'Pegasus is not a winged horse' should be false in such a world-, it does not seem clear at all which truth value -if any- should be given to such statements in a world, like the actual, in which Pegasus does not exist.

§2 Hence, it seems that to make the notion of a possible world scientifically useful, some sort of refinement of the intuitive notion should be required, and this is precisely what has been done by the contemporary semantics of modal logic developed by Kripke and others by relativizing such a notion.

In the semantics of modal logic we say that a statement ϕ is necessarily true -or simply, necessary- in a given world W if and only if ϕ is true in every world W^* possible relative to W . As is well known, the semantics of the different systems of modal logic differ from each other essentially in the properties constitutive of the notion of possible world relative to W .

On the other hand, the semantics of modal logic takes for granted the intuitively very plausible assumption that individuals can exist in some possible worlds but not exist in others. Although Pegasus does not exist in the actual world, it could have existed, and although Quine exists in the actual world, he could have not existed. It also seems to correspond to our intuitive notion of a possible world that in some possible world Quine is not a philosopher, but, e.g., a politician, and that the statement 'Aristotle is the teacher of Alexander the Great' be true in some possible worlds - e.g., in the actual-, but false in other possible worlds in which Aristotle and Alexander both exist, whereas the statement '2 is a square root of 4' is true in every possible world in which the numbers 2 and 4 both exist. The semantics of modal logic tries to do justice to such intuitions.

In the case of a statement containing denotative expressions, such a statement is necessary if it is true in every possible world in which the objects referred to by such expressions exist. Hence, whereas the statements '2 is a square root of 4' and 'Pegasus is a winged horse' seem to be necessary -according to the foregoing characterization-, the statements 'Quine is a philosopher' and 'Aristotle is the teacher of Alexander the Great' seem to be only contingently true.

Nevertheless, the 'material adequacy' of the semantics of modal logic is far from evident.

§3 Intuitively, it seems clear that an individual, e.g., Frege, exists in some possible worlds and not in others, and the semantics of modal logic takes such an intuition into account. Nevertheless, a statement that asserts that Frege exists, e.g., $(\exists x)(x=\text{Frege})$ is (logically) necessary, since it is true in every possible world in which the objects referred to by denotative expressions occurring in the statement exist. Thus, such a statement is necessary, since it is true in every possible world in which Frege exists. A similar unpleasant consequence can be obtained if we consider the negation of such a statement, namely, the statement $\neg(\exists x)(x=\text{Frege})$. Intuitively, this last statement should be possible, since there seem to exist possible worlds in which Frege does not exist. However, $\neg(\exists x)(x=\text{Frege})$ is false in every possible world in which Frege exists, and since according to the definition of logical necessity, those are the only possible worlds relevant for the truth or falsehood of sentences containing denotative expressions, $\neg(\exists x)(x=\text{Frege})$ is not only false but necessarily false.

The situation turns out still worse when we ask about the conditions under which a statement like $\neg(\exists x)(x=\text{Frege})$ would be necessary. On the basis of Kripke's characterization of necessity for statements containing denotative expressions, the statement $\neg(\exists x)(x=\text{Frege})$, as every other statement that contains denotative expressions, would be necessary if and only if it is true in every possible world in which the objects referred to by the denotative expressions occurring in it exist. Hence, $\neg(\exists x)(x=\text{Frege})$ would be necessary if and only if it is true in every possible world in which Frege exists. But this is, of course, unacceptable, since in that case $(\exists x)(x=\text{Frege})$ and $\neg(\exists x)(x=\text{Frege})$ would have the same necessity conditions, i.e., they would be necessary under the same circumstances, and this is really absurd.³ Therefore, the characterization given by Kripke of the necessity of statements that contain denotative expressions cannot be applied consistently to existential statements. Thus, we have either to abandon such a characterization or to limit its applicability in some way that excludes existential statements. On the

³ Moreover, since $\neg(\exists x)(x=\text{Frege})$ is the negation of $(\exists x)(x=\text{Frege})$, the following contradiction would follow: $\neg(\exists x)(x=\text{Frege})$ is necessary (or necessarily true) if and only if it is true in every possible world in which Frege exists if and only if $(\exists x)(x=\text{Frege})$ is true in every possible world in which Frege exists if and only if $\neg(\exists x)(x=\text{Frege})$ is false in every possible world in which Frege exists if and only if $\neg(\exists x)(x=\text{Frege})$ is necessarily false.

other hand, if following our intuitions, we consider a statement like $\neg(\exists x)(x=\text{Frege})$ as true in every possible world in which Frege does not exist, then our characterization of the truth conditions of such a statement would differ from -and even be inconsistent with- that given by Kripke for the (rest of the) statements containing denotative expressions. But this is also unacceptable. (It should be clear that if we substitute 'true' for 'necessary' in the foregoing argumentation, a similar unwelcomed conclusion can be obtained, namely: $\neg(\exists x)(x=\text{Frege})$ is true if and only if $(\exists x)(x=\text{Frege})$ is true.)

§4 The situation is also particularly unpleasant if we consider an object, like Pegasus, not existing in the actual world, but such that it seems reasonable to suppose that it exists in some possible world. Let W be a possible world in which Pegasus exists. In such a world the statement $(\exists x)(x=\text{Pegasus})$ is not only true, but necessarily true, since it is true in every possible world in which Pegasus exists. Therefore, $\Box(\exists x)(x=\text{Pegasus})$ is true in W . Hence, the actual world is not a possible world relative to W , since Pegasus does not exist in the actual world. But since W was arbitrary, there does not exist any possible world W^* in which $(\exists x)(x=\text{Pegasus})$ is true and such that our world is a possible world relative to W^* .

If the relation of possibility (or accessibility) relative to a world W were symmetric, as is the case in the semantics of S5 and the so-called Brouwer system, no world W^* in which Pegasus exists would be possible relative to the actual world. But this is clearly counterintuitive, since we can imagine a world -even one very similar to the actual world- in which Pegasus exists. Of course, the relation of possibility relative to a world W need not be symmetric -e.g., it is not symmetric in S4. Our difficulties, however, have not come to an end.

Let us suppose that a world W in which Pegasus exists is possible relative to the actual world (even though the actual world is not possible relative to W). In W $(\exists x)(x=\text{Pegasus})$ is not only true, but necessarily true, since it is true in every possible world in which Pegasus exists. But Pegasus does not exist in the actual world. Thus, it seems reasonable to consider the statement $(\exists x)(x=\text{Pegasus})$ as false in the actual world, and its negation, namely, $\neg(\exists x)(x=\text{Pegasus})$ as true. We would then have the counterintuitive situation, in which the statement $(\exists x)(x=\text{Pegasus})$ is false in the actual world, but nevertheless necessarily true in a world W possi-

ble relative to the actual world. Analogously, although the statement $\neg(\exists x)(x=\text{Pegasus})$ is true in the actual world, it is necessarily false in W , which is a possible world relative to the actual world. It would also be counterintuitive not to assign any truth value to such statements in the actual world, since it seems intuitively clear that $(\exists x)(x=\text{Pegasus})$ is false in the actual world, whereas $\neg(\exists x)(x=\text{Pegasus})$ is true in the actual world. The only way out seems to be to consider any such world W in which Pegasus exists as not possible relative to the actual world. Since, on the other hand, the actual world is not possible relative to any world W in which Pegasus exists, our actual world has to be incommunicated from every world in which Pegasus exists. Thus, if W is a world in which Pegasus exists -and, intuitively, such a world seems to be possible-, then W is not (logically) possible relative to the actual world, nor the actual world is (logically) possible relative to W . Since the situation remains essentially the same if we substitute for Pegasus any other non-existing but logically consistent object⁴, we can conclude that our actual world and any other world in which there exists an object that does not exist in the actual world, are not logically possible relative to each other. But this conclusion is hardly acceptable to a modal logician (and to almost anyone).

Since $(\exists x)$ can always be defined in terms of $(\forall x)$ and \neg , we leave it to the reader to judge to what extent the legitimacy of a semantics of quantified modal logic is affected by such an argumentation.⁵

Universidad de Puerto Rico

⁴ I.e., an object to which no pair of logically incompatible properties is assigned.

⁵ This paper has a rather strange history. It was written -in essentially the present version- during the winter of 1982-83 (either in December 1982 or in February 1983) while I was in Belgium writing a book on Frege in Spanish. Since I thought that there must be a hidden error in my argumentation and since the paper deviated from my main interests, I put it aside and continued working on other issues. Sometime later I lost all the copies of the paper and it was not until almost a decade later that my mother found them in her house in 1994. I have decided to finally publish it with only minor changes.

REFERENCES

- (1) Kripke, S. "Semantical Considerations on Modal Logic", in *Acta Philosophica Fennica* **16**, 1963, pp. 83-94.
- (2) Kripke, S. *Naming and Necessity*, 1970, reprint, Oxford, 1980
- (3) Rosado Haddock, G. E. "Necessità a posteriori e Contingenze a priori in Kripke; Alcune Note Critiche", in *Nominazione* **2**, June 1982, pp. 205-219.